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Dark matter in the supersymmetric radiative seesaw model with an anomalous U(1) symmetry

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Abstract

The existence of an anomalous U(1) symmetry is shown to play a crucial role in the supersymmetric radiative seesaw model for neutrino masses. It explains the smallness of some couplings related to neutrino mass generation in a favorable way in addition to cause the hierarchical structure of Yukawa couplings of quarks and leptons. If it is spontaneously broken to a Z_2 subgroup, this Z_2 symmetry can make a lifetime of the lightest field with its odd parity extremely long. Thus, the model has an additional dark matter candidate other than the lightest neutralino, which appears in the R -parity conserved MSSM. We discuss the nature of dark matter by taking account of its relation to the neutrino mass generation and the lepton flavor violation.

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1 Introduction

The recent astrophysical observations [1] and neutrino oscillation experiments [2] require that the standard model (SM) should be extended so as to include dark matter (DM) [3] and small neutrino masses. A radiative seesaw model proposed by Ma [4] is a simple and interesting possibility among such extensions.³ In this model the existence of DM is intimately related to the neutrino mass generation. The combined study of the neutrino oscillation data, the lepton flavor violating processes such as $\mu \rightarrow e\gamma$ and the DM relic abundance can give strong constraints on the model [5, 6]. On the other hand, the recently reported cosmic ray anomaly [12, 13] attracts a lot of attention as a target of the DM physics. Various work relevant to this has been done on the basis of the annihilation of DM [7, 14, 15, 16, 17] and also the decay of DM [18, 19, 20, 21, 22]. If we impose that this neutrino mass model should explain this anomaly, it could bring the model additional valuable information not only on the mass and the interaction of DM but also on the lepton flavor structure [17]. Although the model shows interesting and promising features on phenomenology, the model cannot give any answer to the hierarchy problem unfortunately. Previous models are usually constructed in the nonsupersymmetric framework. Thus, the detailed study on the supersymmetrization of the model is one of the remaining subjects relevant to the model.

The supersymmetric extension of the model has been considered in [23, 24].⁴ Two DM candidates appear in these extensions as long as R -parity conservation is assumed. The existence of two kinds of Z_2 symmetry, that is, R -parity and Z_2 which forbids tree-level neutrino masses, guarantees the stability of these two DM candidates. In that case both of them contribute to the expected DM relic abundance.⁵ However, if one of them is unstable but has an longer life time than the age of universe, this decaying DM may cause the additional contribution cosmic ray, which brings anomalous excess of the charged particle flux over the expected background. Such a possibility has been studied in a supersymmetrized model in [24], where Z_2 is assumed to be weakly broken by the anomaly effect. Since the anomaly induced interaction is strongly suppressed, it causes

³There are a lot of study for the DM nature in radiative neutrino mass generation models and other phenomenological features of such models [5, 6, 7, 8, 9, 10, 11].

⁴A relevant supersymmetric model is considered in a different context in [25].

⁵Multi-component DM models have been considered in several contexts [26].

the DM decay but its life time can be long enough.

In this paper, we consider a modification of the original one given in [24] by introducing the an anomalous $U(1)$ gauge symmetry. Spontaneous breaking of this symmetry causes the weakly broken Z_2 symmetry at low energy regions naturally in addition to the conserved R -parity. It makes us possible to understand the smallness of some couplings through the Froggatt-Nielsen mechanism [27, 28], which is required to generate the small neutrino masses.⁶ Simultaneously, hierarchical masses of quarks and charged leptons are also explained by the same origin. The model also shows another interesting features relevant to the DM phenomenology. We give its analysis which includes the cosmic ray anomaly expected in this model, the characteristic gamma predicted in the DM radiative decay and the direct search of the stable DM through the elastic scattering with nuclei.

The paper is organized as follows. In section 2 we give the low energy effective model with an anomalous $U(1)$ gauge symmetry. Spontaneous breaking of this symmetry gives the Z_2 symmetry which is relevant to the neutrino mass generation. The neutrino mass and mixing, the lepton flavor violating processes and the DM relic abundance are studied in this framework. In section 3 the nature of DM is discussed from a view point of both direct and indirect searches. Section 4 is devoted to the summary.

2 A supersymmetric extension

The radiative seesaw model proposed in [4] is an extension of the SM with three right-handed neutrinos and an inert doublet scalar.⁷ It has no coupling with quarks and no vacuum expectation value (VEV). Although the model has interesting phenomenological features as discussed in [5, 6, 8], it can not give an answer to the hierarchy problem. Thus, it is worthy to consider the supersymmetric extension of the model and to study phenomenology associated to it. The supersymmetrization of the model requires to introduce inert doublet chiral superfields η_u and η_d and also a singlet chiral superfield ϕ to the minimal supersymmetric SM (MSSM) [24]. They play the similar role as the inert doublet scalar in the original nonsupersymmetric model and bring required terms in the

⁶ An anomalous $U(1)$ symmetry has been considered in various phenomenological contexts [29]. In a radiative seesaw mechanism, it is also considered in [30]. However, the present model is different from it in the nature of dark matter and also the R -parity conservation.

⁷The origin of the second doublet is discussed in [31].

Ψ	Q_i	\bar{U}_i	\bar{D}_i	L_i	\bar{E}_i	H_u	H_d
$SU(2)_L \times U(1)_Y$	$(\underline{2}, \frac{1}{6})$	$(\underline{1}, \frac{-2}{3})$	$(\underline{1}, \frac{1}{3})$	$(\underline{2}, \frac{-1}{2})$	$(\underline{1}, 1)$	$(\underline{2}, \frac{1}{2})$	$(\underline{2}, \frac{-1}{2})$
R	—	—	—	—	—	+	+
X	$2n_{Q_i}$	$2n_{U_i}$	$2n_{D_i}$	$2n_{L_i}$	$2n_{E_i}$	$2n_{H_u}$	$2n_{H_d}$
Z_2	+	+	+	+	+	+	+
Ψ	\bar{N}_i	η_u	η_d	ϕ	Σ_+	Σ_-	
$SU(2)_L \times U(1)_Y$	$(\underline{1}, 0)$	$(\underline{2}, \frac{1}{2})$	$(\underline{1}, \frac{-1}{2})$	$(\underline{1}, 0)$	$(\underline{1}, 0)$	$(\underline{1}, 0)$	
R	+	—	—	—	+	+	
X	$2n_{N_i} + 1$	$2n_{\eta_u} + 1$	$2n_{\eta_d} + 1$	$2n_{\phi} + 1$	$2n_+$	-2	
Z_2	—	—	—	—	+	+	

Table 1: Matter contents and their quantum number. X represents the charge of the anomalous $U(1)_X$ and each n_Ψ is an integer. Z_2 is a remnant symmetry of $U(1)_X$ caused by the symmetry breaking due to $\langle \Sigma_\pm \rangle \neq 0$.

Lagrangian for the neutrino mass generation. As the origin of the Z_2 symmetry which forbids tree level neutrino masses, we suppose the existence of an anomalous $U(1)_X$ gauge symmetry. We assume that the R -parity is conserved. Matter contents of the model and their relevant quantum numbers are summarized in Table 1.

2.1 A low energy effective model

First, we discuss spontaneous breaking of the anomalous $U(1)_X$ symmetry at a high energy scale, which brings the low energy effective theory. Hierarchical couplings and masses are found to be generated in the low energy theory through this breaking. The vacuum is expected to be determined as the flat direction of the D -term of $U(1)_X$. The D -term for relevant hidden fields is given by

$$V_D = \frac{g_X^2}{2} \left[(2n_\phi + 1)|\phi|^2 + 2n_+|\Sigma_+|^2 - 2|\Sigma_-|^2 + \xi_X \right]^2. \quad (1)$$

ξ_X is the $U(1)_X$ Fayet-Iliopoulos D -term. It is expressed in string models as [32]

$$\xi_X = \frac{\text{Tr} X}{192\pi^2} g_X^2 M_{\text{pl}}^2 \equiv \delta_{\text{GS}} g_X^2 M_{\text{pl}}^2, \quad (2)$$

where M_{pl} is the reduced Planck mass and X stands for the anomalous $U(1)_X$ charge of the fields. On the other hand, the lowest order superpotential for these fields can be

written as

$$W_h = \frac{c_+}{M_{\text{pl}}^{n_+-2}} \Sigma_+ \Sigma_-^{n_+} + \frac{c_\phi}{M_{\text{pl}}^{2n_\phi}} \phi^2 \Sigma_-^{2n_\phi+1}, \quad (3)$$

where c_+ and c_ϕ are considered as real constants. If we suppose supersymmetry breaking in the hidden sector, supersymmetry breaking terms appear in the scalar potential of the hidden sector. The scalar potential derived from W_h may be written as

$$\begin{aligned} V_F = & \frac{c_+^2}{M_{\text{pl}}^{2(n_+-2)}} |\Sigma_-|^{2n_+} + \frac{4c_\phi^2}{M_{\text{pl}}^{4n_\phi}} |\phi \Sigma_-^{2n_\phi+1}|^2 + \frac{c_+^2 n_+^2}{M_{\text{pl}}^{2(n_+-2)}} |\Sigma_+ \Sigma_-^{n_+-1}|^2 \\ & + \frac{c_\phi^2 (2n_\phi + 1)^2}{M_{\text{pl}}^{4n_\phi}} |\phi^2 \Sigma_-^{2n_\phi}|^2 + \left(\frac{c_+ n_+ c_\phi (2n_\phi + 1)}{M_{\text{pl}}^{n_++2n_\phi-2}} (\Sigma_+ \Sigma_-^{n_+-1})^* \phi^2 \Sigma_-^{2n_\phi} + \text{h.c.} \right), \\ & - F_1 \Sigma_-^2 - F_2 \Sigma_+^2, \end{aligned} \quad (4)$$

where F_1 and F_2 represent the VEVs of F -components of some chiral superfields in the hidden sector. They bring the supersymmetry breaking of $O(10^{11})$ GeV in the hidden sector and also induce the soft terms of $O(1)$ TeV through the gravity mediation in the observable sector.

Here we assume that the VEVs of Σ_\pm and ϕ are real, for simplicity. Then, the minimization of the potential $V = V_D + V_F$ along the D -flat direction suggests that there exists a vacuum defined by

$$\langle \Sigma_- \rangle \simeq \sqrt{\frac{\xi_X}{2}} \gg \langle \Sigma_+ \rangle, \quad \langle \phi \rangle = 0, \quad (5)$$

as long as c_+ is sufficiently suppressed. For example, if $\text{Tr} X \sim 150/g_X^2$ is satisfied, $\langle \Sigma_- \rangle \sim 0.2 M_{\text{pl}}$ is expected. Moreover, $\langle \Sigma_+ \rangle \simeq 10^{-4} M_{\text{pl}}$ is also expected for sufficiently suppressed values of c_+ such as $O(10^{-7})$. Although this kind of vacuum can be realized only for the finely tuned parameters, we assume it in the following study.

If we adopt this vacuum to fix the low energy effective theory, the superpotential invariant under the imposed symmetry is obtained with the effectively induced parameters such as

$$\begin{aligned} W = & h_{ij}^U Q_i \bar{U}_j H_u + h_{ij}^D Q_i \bar{D}_j H_d + h_i^E L_i \bar{E}_i H_d + \mu_H H_u H_d, \\ & + h_{ij}^N L_i \bar{N}_j \eta_u + \lambda_u \eta_u H_d \phi + \lambda_d \eta_d H_u \phi + \mu_\eta \eta_u \eta_d + \frac{1}{2} M_i \bar{N}_i \bar{N}_i + \frac{1}{2} \mu_\phi \phi^2. \end{aligned} \quad (6)$$

The invariance of each term under $R \times U(1)_X$ is guaranteed since the effective couplings and masses are generated through the VEVs $\langle \Sigma_\pm \rangle$ as shown below (see also Appendix

A). The MSSM superpotential is contained in the first line, while the second line includes additional terms to the MSSM. Yukawa couplings for the charged leptons and the mass matrix for the right-handed neutrinos are supposed to be flavor diagonal.⁸

The effective parameters in this superpotential are induced from higher order invariant interaction terms which contain a suitable number of Σ_{\pm} . The dominant contribution are determined by the lowest order term of the following form:

$$\begin{aligned} h_{ijk} &= y_{ijk} \left(\frac{\langle \Sigma_{\pm} \rangle}{M_{\text{pl}}} \right)^{n_{ijk}}, & n_{ijk} &= -\frac{X_i + X_j + X_k}{X_{\Sigma_{\pm}}} \quad \text{for } h_{ijk} \Psi_i \Psi_j \Psi_k, \\ \mu_{ij} &= y_{ij} M_{\text{pl}} \left(\frac{\langle \Sigma_{\pm} \rangle}{M_{\text{pl}}} \right)^{n_{ij}}, & n_{ij} &= -\frac{X_i + X_j}{X_{\Sigma_{\pm}}} \quad \text{for } \mu_{ij} \Psi_i \Psi_j. \end{aligned} \quad (7)$$

The original coupling constants y_{ijk} and y_{ij} in the nonrenormalizable interaction terms are considered to be values of $O(1)$. If the singlet scalars Σ_{\pm} obtain the VEVs as discussed above, these VEVs cause hierarchical structure in the Yukawa couplings of quarks and leptons, and also suppress several parameters in the superpotential as found from eqs. (5) and (7). In fact, as long as suitable $U(1)_X$ charges are assigned to quarks and leptons, we find that the hierarchical mass eigenvalues and mixing are generated via Froggatt-Nielsen mechanism [27]. In the similar way, the Yukawa couplings $\lambda_{u,d}$ are largely suppressed, and also μ_{η} and M_i can take values of $O(1)$ TeV. These parameters are relevant to the neutrino mass generation and their values can be favorable for it as seen in the next part. We give more detailed discussion by giving such concrete examples for the charge assignment in the Appendix A. We also find that $\langle \Sigma_{\pm} \rangle$ breaks the anomalous $U(1)_X$ symmetry spontaneously to its Z_2 subgroup. This is obvious from the charge assignment shown in Table 1. This Z_2 symmetry plays a crucial role in the DM phenomenology.

Here we also note an interesting point related to the anomaly induced interaction. We have $L_i \eta_u$ as an only renormalizable operator which breaks the $U(1)_X$ invariance but is invariant under the SM gauge symmetry and the R -parity. It is not included in the low energy superpotential (6) since we can not make it $U(1)_X$ invariant by multiplying any number of Σ_{\pm} . However, it can be $U(1)_X$ invariant if it is accompanied by a dilaton chiral superfield S . As is well known, if the anomalous $U(1)_X$ gauge transformation $V_X \rightarrow V_X + i(\Lambda - \Lambda^\dagger)/2$ is associated with the shift of the dilaton field such as

$$S \rightarrow S + i\delta_{\text{GS}}\Lambda, \quad (8)$$

⁸This can be justified as long as the relevant original couplings y_{ijk} and y_{ij} in eq. (7) are flavor diagonal.

the anomaly cancellation for $U(1)_X$ is completed [33, 34]. Noting this fact, we observe that the following non-perturbative superpotential is also invariant under the imposed symmetry [34]:⁹

$$\begin{aligned} W_{\text{np}} &= c_i M_{\text{pl}} e^{-b_i L_i \eta_u}, \\ b_i &= \frac{(2n_i^L + 2n_{\eta_u} + 1)}{\delta_{\text{GS}}} S \sim \frac{192\pi^2(2n_i^L + 2n_{\eta_u} + 1)}{\text{Tr}X} \end{aligned} \quad (9)$$

where we use $\langle S \rangle$ and c_i are constants of $O(1)$. The expression for b_i shows that it is determined by the $U(1)_X$ charge of the field contents of the model including the hidden sector. We also note that this term breaks the remnant Z_2 symmetry very weakly. The scale of its violation is determined by both the value of $\text{Tr}X$ and the charges of L_i and η_u .

Soft supersymmetry breaking terms associated with the superpotential W and W_{np} are introduced as follows,

$$\begin{aligned} \mathcal{L}_{SB} &= -\tilde{m}_{\eta_u}^2 \tilde{\eta}_u^\dagger \tilde{\eta}_u - \tilde{m}_{\eta_d}^2 \tilde{\eta}_d^\dagger \tilde{\eta}_d - \tilde{m}_{\tilde{N}}^2 \tilde{N}_i^\dagger \tilde{N}_i - \tilde{m}_{\tilde{\phi}}^2 \tilde{\phi}^\dagger \tilde{\phi} \\ &\quad + A(h_{ij}^N \tilde{L}_i \tilde{N}_j \tilde{\eta}_u + \lambda_u \tilde{\eta}_u H_d \tilde{\phi} + \lambda_d \tilde{\eta}_d H_u \tilde{\phi} + \text{h.c.}) \\ &\quad - B \left(\mu_H H_u H_d + \mu_\eta \tilde{\eta}_u \tilde{\eta}_d + \frac{1}{2} \mu_\phi \tilde{\phi}^2 + \frac{1}{2} M_i \tilde{N}_i^2 + c_i M_{\text{pl}} e^{-b_i \tilde{L}_i \tilde{\eta}_u} + \text{h.c.} \right), \end{aligned} \quad (10)$$

where the additional part to the MSSM is listed alone. The scalar component is represented by putting a tilde on the character of the corresponding chiral superfield except for the ordinary Higgs chiral superfields H_u and H_d . Universality of the soft supersymmetry breaking A - and B -parameters is assumed to be satisfied here. Moreover, we confine our consideration to the case in which the soft scalar masses for all scalar partners of quarks and leptons are flavor diagonal and universal. They are denoted by m_0 in the following analysis and we assume $A = B = m_0$, for simplicity.

We should remind the reader that the soft scalar masses have the contributions from the anomalous $U(1)_X$ D -term such as

$$\Delta \tilde{m}_i^2 = g_X^2 X_i (C_h + 2n_{H_u} \langle H_u^0 \rangle^2 + 2n_{H_d} \langle H_d^0 \rangle^2), \quad (11)$$

where C_h stands for the remnant contribution from the hidden sector. Although these are flavor dependent, we confine our study to the case $m_0^2 \gg \Delta \tilde{m}_i^2$, which may be realized on

⁹The effect of the appearance of this term has been studied in the case of discrete symmetry in [24].

the vacuum with a finely tuned C_h . In such a case there is no new dangerous origin for the flavor changing neutral current (FCNC) caused by the soft supersymmetry breaking terms. In the following study the effective parameters in eq. (6) and also the soft supersymmetry breaking parameters in eq. (10) are treated as real except for h_{ij}^N , for simplicity.

2.2 Neutrino mass and mixing

Neutrino masses are generated through the one-loop diagram shown in Fig. 1 when the Higgs doublet scalars H_u^0 and H_d^0 obtain the VEVs. As found from eq. (40), since λ_u and λ_d are very small, the mixing between $(\tilde{\eta}_u^{0*}, \tilde{\eta}_d^0)$ and $(\tilde{\phi}^*, \tilde{\phi})$ can be treated as an insertion in the calculation of these diagrams with good accuracy. The mass matrices for $(\tilde{\eta}_u^{0*}, \tilde{\eta}_d^0)$ and $(\tilde{\phi}^*, \tilde{\phi})$ are written as

$$\mathcal{M}_{\eta^0} = \begin{pmatrix} \bar{m}_{\eta_u}^2 & B\mu_\eta \\ B\mu_\eta & \bar{m}_{\eta_d}^2 \end{pmatrix}, \quad \mathcal{M}_\phi = \frac{1}{2} \begin{pmatrix} \bar{m}_\phi^2 & B\mu_\phi \\ B\mu_\phi & \bar{m}_\phi^2 \end{pmatrix} \quad (12)$$

where $\bar{m}_{\eta_{u,d}}^2 \simeq m_0^2 + \mu_\eta^2 + \lambda_{u,d}^2 v_{d,u}^2$ and $\bar{m}_\phi^2 \simeq m_0^2 + \mu_\phi^2 + \lambda_u^2 v_d^2 + \lambda_d^2 v_u^2$. If we define the mass eigenstates of these mass matrices by

$$\begin{pmatrix} \tilde{\eta}_+ \\ \tilde{\eta}_- \end{pmatrix} = \begin{pmatrix} \cos \theta_\eta & \sin \theta_\eta \\ -\sin \theta_\eta & \cos \theta_\eta \end{pmatrix} \begin{pmatrix} \tilde{\eta}_u^{0*} \\ \tilde{\eta}_d^0 \end{pmatrix}, \quad \begin{pmatrix} \tilde{\phi}_+ \\ \tilde{\phi}_- \end{pmatrix} = \begin{pmatrix} \cos \theta_\phi & \sin \theta_\phi \\ -\sin \theta_\phi & \cos \theta_\phi \end{pmatrix} \begin{pmatrix} \tilde{\phi}^* \\ \tilde{\phi} \end{pmatrix}, \quad (13)$$

the mass eigenvalues and the mixing angles can be written as

$$m_{\eta^\pm}^2 = \frac{1}{2} \left(\bar{m}_{\eta_u}^2 + \bar{m}_{\eta_d}^2 \pm \sqrt{(\bar{m}_{\eta_u}^2 - \bar{m}_{\eta_d}^2)^2 + 4B^2\mu_\eta^2} \right), \quad \tan 2\theta_\eta = \frac{2B\mu_\eta}{\bar{m}_{\eta_d}^2 - \bar{m}_{\eta_u}^2},$$

$$m_{\phi^\pm}^2 = \bar{m}_\phi^2 \pm B\mu_\phi, \quad \theta_\phi = \frac{\pi}{4}. \quad (14)$$

We find that the scalar superpartners $(\tilde{N}_i^*, \tilde{N}_i)$ of \bar{N}_i satisfy the same relations as the ones of $(\tilde{\phi}^*, \tilde{\phi})$ as shown above. Their mass eigenvalues $M_{i\pm}^2$ can be read off from the expression for $m_{\phi^\pm}^2$ by replacing μ_ϕ and \bar{m}_ϕ^2 with M_i and $m_0^2 + M_i^2$, respectively.

We can calculate the neutrino masses generated through the one-loop diagrams by using these. Since μ_ϕ^2 is expected to be larger than μ_η^2 and m_0^2 , we find that the dominant contribution is caused by the diagram (a) in Fig. 1 and it is estimated as

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{\lambda_u \lambda_d v_u v_d \sin 2\theta_\eta}{16\pi^2} \sum_{i=1}^3 h_{\alpha i} h_{\beta i} M_i \left(g(M_i, m_{\eta+}) \cos^2 \theta_\eta - g(M_i, m_{\eta-}) \sin^2 \theta_\eta - f(M_i, m_{\eta+}, m_{\eta-}) \cos 2\theta_\eta \right), \quad (15)$$

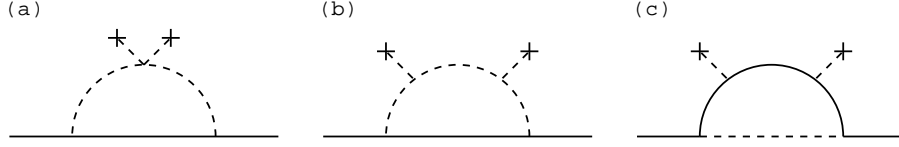


Fig. 1 One-loop diagrams contributing to the neutrino masses. Two diagrams (b) and (c) have an internal line of the scalar $\tilde{\phi}$ or its fermionic partner, respectively.

where f and g are defined as

$$f(m_a, m_b, m_c) = \frac{m_a^2 m_b^2 \ln(m_b^2/m_a^2) + m_b^2 m_c^2 \ln(m_c^2/m_b^2) + m_c^2 m_a^2 \ln(m_a^2/m_c^2)}{(m_c^2 - m_a^2)(m_a^2 - m_b^2)(m_b^2 - m_c^2)},$$

$$g(m_a, m_b) = \frac{m_b^2 - m_a^2 + m_a^2 \ln(m_a^2/m_b^2)}{(m_b^2 - m_a^2)^2}. \quad (16)$$

Other two diagrams (b) and (c) with the component of ϕ as an internal line are expected to bring the subdominant contributions. We give the explicit expressions of their contributions in the Appendix B. The universal soft supersymmetry breakings make the situation simple since $\theta_\eta = \pi/4$ is satisfied. If M_i and $\bar{m}_{\eta_{u,d}}$ have the values of $O(1)$ TeV, the mass eigenvalues of neutrinos must be controlled by a small parameter $\lambda_u \lambda_d$ of $O(10^{-8})$. As long as the anomalous $U(1)_X$ charge is assigned suitably, these can be naturally realized as found from the examples given in the Appendix A.

Here we focus our attention to the flavor structure of the neutrino Yukawa couplings¹⁰ [6, 7]

$$h_{ei}^N = 0, \quad h_{\mu i}^N = h_{\tau i}^N \equiv h_i \quad (i = 1, 2), \quad h_{e3}^N = h_{\mu 3}^N = -h_{\tau 3}^N \equiv h_3. \quad (17)$$

This flavor structure is consistent with the $U(1)_X$ invariance as long as all of y_{ij}^N take the same values except for y_{ei}^N , which should be assumed to be $y_{ei}^N = 0$ for $i = 1, 2$ (see Appendix A). An interesting point of this flavor structure is that the neutrino mass matrix in eq.(15) takes the following simple form:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} (h_1^2 \Lambda_1 + h_2^2 \Lambda_2) + \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} h_3^2 \Lambda_3, \quad (18)$$

¹⁰This structure is chosen adhoc here since it has several interesting features as shown below.

where Λ_i fixes the mass scale for the neutrino masses as follows,¹¹

$$\Lambda_i = \frac{\bar{\lambda} v^2 M_i}{32\pi^2} \left(g(M_i, m_{\eta+}) - g(M_i, m_{\eta-}) \right), \quad \bar{\lambda} \equiv \frac{\lambda_u \lambda_d \tan \beta}{1 + \tan^2 \beta}, \quad (19)$$

where $\sin \beta = v_u/v$ and $\cos \beta = v_d/v$. This mass matrix automatically derives the tri-bimaximal mixing, which is favored by the neutrino oscillation data. In fact, it is easily checked that the MNS matrix for this neutrino mass model is given by

$$U_{MNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}, \quad (20)$$

where Majorana phases $\alpha_{1,2}$ are expressed as

$$\alpha_1 = \varphi_3, \quad \alpha_2 = \frac{1}{2} \tan^{-1} \left(\frac{|h_1|^2 \Lambda_1 \sin 2\varphi_1 + |h_2|^2 \Lambda_2 \sin 2\varphi_2}{|h_1|^2 \Lambda_1 \cos 2\varphi_1 + |h_2|^2 \Lambda_2 \cos 2\varphi_2} \right) \quad (21)$$

by using $\varphi_i = \arg(h_i)$. Here it should be reminded that Majorana phases $\alpha_{1,2}$ do not affect the neutrino oscillations.

Taking account of the nature discussed above and also the fact that one of the eigenvalues of the mass matrix (18) is zero, the remaining mass eigenvalues are found to be equal to $\sqrt{\Delta m_{\text{atm}}^2}$ and $\sqrt{\Delta m_{\text{sol}}^2}$. This is required for the explanation of neutrino oscillation data. Thus, we find that the model parameters should satisfy the relations

$$|h_1^2 \Lambda_1 + h_2^2 \Lambda_2| \simeq \frac{\sqrt{\Delta m_{\text{atm}}^2}}{2}, \quad |h_3^2 \Lambda_3| \simeq \frac{\sqrt{\Delta m_{\text{sol}}^2}}{3}. \quad (22)$$

The neutrino Yukawa couplings h_i and the right-handed neutrino masses M_i should correlate each other so as to satisfy these relations. Phenomenological study of the model should be proceeded under these constraints. In the following discussion, we restrict our study to the case with $M_1 \lesssim M_2 < M_3$, which allows us to take $\Lambda_1 \simeq \Lambda_2$.¹² Thus, free parameters relevant to the analysis of DM phenomenology are summarized as

$$M_1, \quad M_3, \quad \bar{\lambda}, \quad \mu_{\eta}, \quad m_0. \quad (23)$$

¹¹If the mass matrix is modified from eq. (15) due to other contributions neglected as the subdominant ones here, the ambiguity is confined into the Λ_i . The MNS matrix is not affected as long as the condition (17) is satisfied.

¹² It may be useful to note that this case has some advantages as discussed in [7, 17], but not only for simplicity.

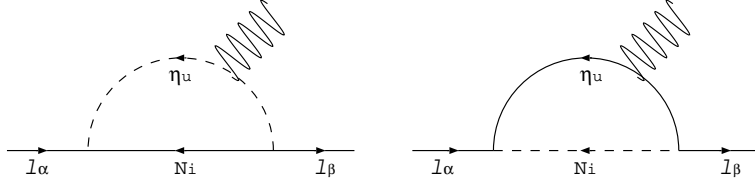


Fig. 2 One-loop diagrams causing the lepton flavor violating processes $\ell_\alpha \rightarrow \ell_\beta \gamma$.

If we suppose fixed the values for M_i , μ_η and \tilde{m}_0 , the conditions (22) determine a value of $\bar{\lambda}\sqrt{h_1^2 + h_2^2}$ and $\bar{\lambda}|h_3|$. A larger $\bar{\lambda}$ gives a smaller value of neutrino Yukawa couplings $\bar{\lambda}\sqrt{h_1^2 + h_2^2}$ and $\bar{\lambda}|h_3|$. This feature becomes crucial when the constraints from lepton flavor violating processes and DM relic abundance are taken into account.

2.3 Constraints from lepton flavor violating processes

Since we suppose the slepton mass matrix is flavor diagonal and universal, there are no new FCNC source in the slepton sector. The FCNC is induced only through the MNS matrix elements which appear in the Higgsino exchange diagrams. Thus, if Higgsinos are much heavier than gauginos, the ordinary contributions to the FCNC caused by the supersymmetric partners are sufficiently suppressed in this model. On the other hand, the extension for the neutrino mass generation induces the one-loop contribution to the lepton flavor violating processes such as $\mu \rightarrow e \gamma$ as in the nonsupersymmetric case. Their diagrams are shown in Fig. 2. Since these diagrams do not need the mixing between η_u and η_d unlike the case of neutrino masses (see Fig. 1), it causes the large contributions to these processes. In that case the dominant contributions to the lepton flavor violating processes are given by these diagrams. They give the constraints on the model, which is the different from the ones in the MSSM. These processes may be used as the probe of the model in the future experiments.

Branching ratio of the lepton flavor violating process $\ell_\alpha \rightarrow \ell_\beta \gamma$ is given as

$$Br(\ell_\alpha^- \rightarrow \ell_\beta^- \gamma) = \frac{3\alpha}{64\pi} \left| \sum_{i=1}^3 h_{\alpha i} h_{\beta i}^* \sum_{a=\pm} \left[\frac{1}{2G_F m_{\eta a}^2} F_2 \left(\frac{M_i^2}{m_{\eta a}^2} \right) + \frac{a}{2G_F \mu_\eta^2} F_2 \left(\frac{M_{ia}^2}{\mu_\eta^2} \right) \right] \right|^2 \times Br(\ell_\alpha^- \rightarrow \ell_\beta^- \bar{\nu}_\beta \nu_\alpha), \quad (24)$$

where $m_{\eta a}$ and M_{ia} are the mass eigenvalues defined in eq. (14) and the statements below

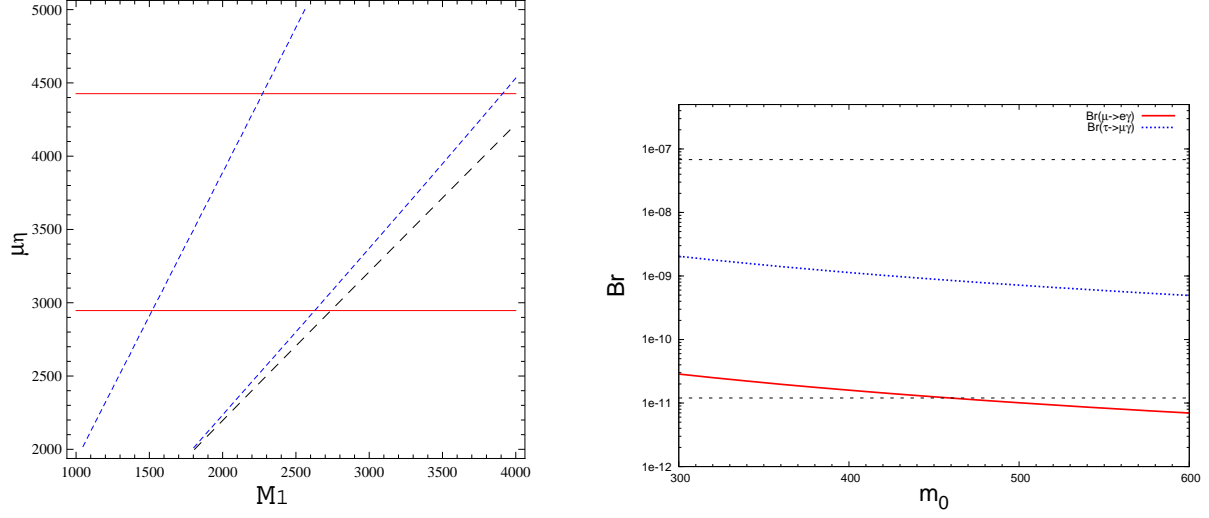


Fig. 3 The left frame shows the contours for $Br(\mu \rightarrow e\gamma)$ (red solid lines) and $Br(\tau \rightarrow \mu\gamma)$ (blue dashed lines) under the conditions (22) imposed by the neutrino oscillation data. A thin black dashed line represents a line for $m_{\eta-} = M_1$. The right frame shows the m_0 dependence of each branching ratio. Thin black dotted lines represent the present experimental bounds for $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$, respectively. We use a GeV unit for M_1 , m_0 and μ_η .

it respectively. The function $F_2(x)$ is defined as

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1 - x)^4}. \quad (25)$$

If we use the assumed flavor structure for the neutrino Yukawa couplings (17), we find that

$$\begin{aligned} Br(\mu \rightarrow e\gamma) &\simeq \frac{3\alpha|h_3|^4}{64\pi} \left[\sum_{a=\pm} \left\{ \frac{1}{2G_F m_{\eta a}^2} F_2\left(\frac{M_3^2}{m_{\eta a}^2}\right) + \frac{a}{2G_F \mu_\eta^2} F_2\left(\frac{M_{3a}^2}{\mu_\eta^2}\right) \right\} \right]^2, \\ Br(\tau \rightarrow \mu\gamma) &\simeq \frac{0.51\alpha}{64\pi} \left[\sum_{a=\pm} \left\{ \frac{1}{2G_F m_{\eta a}^2} \left((|h_1|^2 + |h_2|^2) F_2\left(\frac{M_1^2}{m_{\eta a}^2}\right) - |h_3|^2 F_2\left(\frac{M_3^2}{m_{\eta a}^2}\right) \right) \right. \right. \\ &\quad \left. \left. + \frac{a}{2G_F \mu_\eta^2} \left((|h_1|^2 + |h_2|^2) F_2\left(\frac{M_{1a}^2}{\mu_\eta^2}\right) - |h_3|^2 F_2\left(\frac{M_{3a}^2}{\mu_\eta^2}\right) \right) \right\} \right]^2 \end{aligned} \quad (26)$$

Now we examine the parameter regions consistent with both the neutrino oscillation data and the lepton flavor violating constraints. By using the above formulas for the lepton flavor violating processes and the neutrino oscillation condition (22), the branching ratio of $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ predicted by the model can be plotted in the (M_1, μ_η) plane if M_3 , m_0 and $\bar{\lambda}$ are settled. It is useful to note that these bounds become more severe by

making $\bar{\lambda}$ smaller generally since a smaller $\bar{\lambda}$ requires larger values for $|h_i|$ as found from eqs. (18) and (19). We fix these values to

$$M_3 = 7.8 \text{ TeV}, \quad m_0 = 0.48 \text{ TeV}, \quad \bar{\lambda} = 1.24 \times 10^{-9} \quad (27)$$

as a typical example. The last one corresponds to $\lambda_u \lambda_d = 10^{-7.7}$ and $\tan \beta = 16$, for example. For a while, we consider the case $\varphi_1 = \varphi_2$ only.

In the left frame of Fig. 3, we show the contours of these branching ratios. Red solid lines represent the contours of $Br(\mu \rightarrow e\gamma) \times 10^{11} = 1.2$ and 0.6 downward, and blue dotted ones are contours of $Br(\tau \rightarrow \mu\gamma) \times 10^8 = 0.1$ and 0.04 rightward. The former one is independent of M_1 as is clear from the expression in eq. (26). $F_2(M_3^2/m_{\eta a}^2)$ becomes smaller for a larger M_3 although the larger M_3 makes the neutrino Yukawa coupling h_3 larger through eq.(22). Thus, this branching ratio can easily satisfy the present experimental bounds by making M_3 large enough. We note that this feature is intimately related to the flavor structure (17) which induces the tri-bimaximal MNS matrix. $Br(\tau \rightarrow \mu\gamma)$ is found to show a different behavior, which is mainly controlled by the masses and the couplings of $\bar{N}_{1,2}$. This figure shows that the present experimental bounds for these processes [35] are satisfied at the wide range of the $m_{\eta-} > M_1$ regions, where $m_{\eta-} = M_1$ is plotted by a thin black dashed line.

In the right frame of Fig. 3, as in the left frame each branching ratio is plotted by a red solid line and a blue dotted line as a function of m_0 for $M_1 = 2.5 \text{ TeV}$ and $\mu_\eta = 4.2 \text{ TeV}$. The present bounds for them are also plotted by the thin black solid and dashed lines, respectively. The figure suggests that the constraint from the $\mu \rightarrow e\gamma$ requires $m_0 \gtrsim 0.45 \text{ TeV}$, although the $\tau \rightarrow \mu\gamma$ gives no constraint on m_0 . From these figures, we find that the model can be easily consistent with both the neutrino oscillation data and the lepton flavor violating constraints for the natural values of parameters as long as they are suitably fixed.

As a related subject, it is useful to note that the similar diagram to Fig. 2 contributes to the electric dipole moment of an electron (EDME) and the muon $g - 2$. Even if the neutrino Yukawa couplings h_{ei}^N are complex, the EDME is not induced since there is no mixing among \bar{N}_i 's. New contributions to the muon $g - 2$ due to the similar diagram to

Fig. 2 are summarized as

$$\delta a_\mu \simeq \frac{m_\mu^2}{2(4\pi)^2} \sum_{a=\pm} \left[\frac{-1}{m_{\eta a}^2} \left\{ (|h_1|^2 + |h_2|^2) F_2 \left(\frac{M_1^2}{m_{\eta a}^2} \right) + |h_3|^2 F_2 \left(\frac{M_3^2}{m_{\eta a}^2} \right) \right\} \right. \\ \left. + \frac{a}{\mu_\eta^2} \left\{ (|h_1|^2 + |h_2|^2) F_2 \left(\frac{M_{1a}^2}{\mu_\eta^2} \right) + |h_3|^2 F_2 \left(\frac{M_{3a}^2}{\mu_\eta^2} \right) \right\} \right]. \quad (28)$$

We estimate it for the allowed parameter sets obtained in the above analysis. The results seem to be smaller by three order of magnitude in comparison with $\delta a_\mu = (30.2 \pm 8.7) \times 10^{-10}$, which is a discrepancy shown by the SM prediction and the value derived by the experiment [36]. This suggests that another origin is required for the explanation of this muon $g - 2$ discrepancy.

2.4 Two dark matter candidate

The model has two types of the DM candidate in general. One of them is the lightest neutralino χ whose stability is guaranteed by the R -parity as in case of the MSSM. The other one is the lightest neutral field with the odd parity of the new Z_2 symmetry, which is the remnant symmetry of the anomalous $U(1)_X$. It corresponds to the lightest neutral state composed of the components of the chiral supermultiplets \bar{N}_i , or $\eta_{u,d}^0$ and ϕ . In the following study, we assume that the singlet fermion ψ_{N_1} (the fermionic component of \bar{N}_1) is the lightest one among these candidates. Since this Z_2 is not an exact symmetry but is weakly broken as shown in eq. (9) by the anomaly effect, the latter candidate ψ_{N_1} is not stable but it could have a long lifetime comparable to the age of the universe. The condition for this possibility is discussed in the next section. If this is the case, the DM relic abundance suggested by the WMAP [1] should be satisfied by these two contributions such as

$$\Omega_\chi h^2 + \Omega_{\psi_{N_1}} h^2 = 0.11. \quad (29)$$

The annihilation of two ψ_{N_1} 's is induced through the t - and u -channel η_u exchange. Final states of such processes are composed of a pair of lepton and antilepton or a pair of slepton and antislepton. The latter final state gives new contributions compared with the non-supersymmetric case as long as $M_1 > \tilde{m}_L (\equiv m_0)$ is satisfied for the slepton mass \tilde{m}_L . If ψ_{N_2} has a mass almost degenerate with the one of ψ_{N_1} and this is the case here, we need to consider the coannihilation effect [37]. Final states of this annihilation are controlled by the flavor structure of the neutrino Yukawa couplings which is shown in eq. (17). On

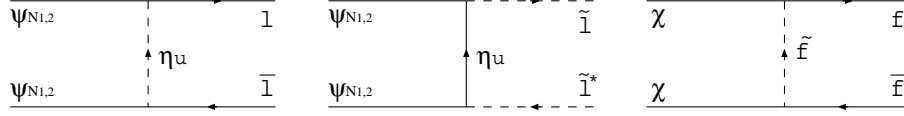


Fig. 4 Diagrams contributing to the t -channel annihilation of the two DM candidates. There are also u -channel diagrams corresponding to these. Although the final state $f\bar{f}$ of the χ annihilation is composed of both $q\bar{q}$ and $\ell\bar{\ell}$, the ψ_{N_1} annihilation contains $\ell\bar{\ell}$ only.

the other hand, the annihilation of two χ 's occurs through various processes depending on its composition, which is determined by both the supersymmetry breaking scenario and the radiative symmetry breaking conditions (see Fig. 4). It has been studied in detail in the MSSM context [3]. Here we do not fix the scenario but just confine our study to the pure bino case by assuming that μ and the masses of Higgs doublet and gauginos are tuned to satisfy the required condition, for simplicity. In this case, the annihilation is expected to occur dominantly through t - and u -channel sfermion exchange.¹³ We estimate the annihilation cross section σv for these DM candidates by expanding it as $\sigma v = a + bv^2$ in powers of their relative velocity v [38].

For the singlet fermion ψ_{N_1} , we need to take account of the coannihilation effect with ψ_{N_2} because of the assumption $M_1 \simeq M_2$. In order to estimate the freeze-out temperature T_f of ψ_{N_1} including the coannihilation case, we follow the procedure given in [37]. We define σ_{eff} and g_{eff} as

$$\begin{aligned}\sigma_{\text{eff}} &= \frac{g_{N_1}^2}{g_{\text{eff}}^2} \sigma_{\psi_{N_1}\psi_{N_1}} + 2 \frac{g_{N_1}g_{N_2}}{g_{\text{eff}}^2} \sigma_{\psi_{N_1}\psi_{N_2}} (1 + \delta)^{3/2} e^{-x\delta} + \frac{g_{N_2}^2}{g_{\text{eff}}^2} \sigma_{\psi_{N_2}\psi_{N_2}} (1 + \delta)^3 e^{-2x\delta}, \\ g_{\text{eff}} &= g_{N_1} + g_{N_2} (1 + \delta)^{3/2} e^{-x\delta},\end{aligned}\tag{30}$$

where internal degrees of freedom of \bar{N}_i are described by g_{N_i} and $\delta \equiv (M_2 - M_1)/M_1$. If we define a_{eff} and b_{eff} by $\sigma_{\text{eff}} v = a_{\text{eff}} + b_{\text{eff}} v^2$, the thermally averaged cross section can be written as $\langle \sigma_{\text{eff}} v \rangle = a_{\text{eff}} + 6b_{\text{eff}}/x$ where $x = M_1/T$. Since $\delta \ll 1$ is supposed here, the second and third terms can bring the important contribution. Using these formulas, the

¹³If the lightest neutralino χ is heavier than Higgs scalars, we need to take account of the t -channel Higgsino exchange process. However, it is expected to be subdominant as long as the Higgsino is heavier than the sfermions, which is assumed throughout this analysis.

effective annihilation cross section is given by

$$\begin{aligned}
(\sigma_{\psi_{N_1}})_{\text{eff}} v &\simeq \frac{Y_s^4}{8\pi} \sum_{a,b=\pm} \frac{M_1^2}{(M_1^2 + m_{\eta a}^2)(M_1^2 + m_{\eta b}^2)} (1 + p_F v^2) \\
&+ \frac{Y_p^4}{96\pi} \sum_{a,b=\pm} \frac{M_1^2 (M_1^4 + m_{\eta a}^2 m_{\eta b}^2)}{(M_1^2 + m_{\eta a}^2)^2 (M_1^2 + m_{\eta b}^2)^2} v^2 \\
&+ \frac{Y_s^4}{8\pi} \frac{M_1^2 \beta}{(\mu_\eta^2 + M_1^2 \beta^2)^2} (\beta^2 + p_S v^2) \\
&+ \frac{Y_p^4}{32\pi} \frac{M_1^2 \beta}{(\mu_\eta^2 + M_1^2 \beta^2)^2} \left(1 + \frac{2\beta^2}{3} - \frac{4\mu_\eta^2 M_1^2 \beta^2}{3(\mu_\eta^2 + M_1^2 \beta^2)^2} \right) v^2, \quad (31)
\end{aligned}$$

where $\beta = \sqrt{1 - (m_0/M_1)^2}$. The mass eigenvalue $m_{\eta a}$ is given in eq. (14). Y_s and Y_p are defined by

$$Y_s^4 = 2|h_1|^2|h_2|^2 \sin^2(\varphi_1 - \varphi_2), \quad Y_p^4 = |h_1|^4 + 2|h_1|^2|h_2|^2 \cos 2(\varphi_1 - \varphi_2) + |h_2|^4. \quad (32)$$

The first and second lines in eq. (31) represent the contributions with the lepton-antilepton final state. The third and forth lines come from the slepton-antislepton final state. We find that there can be s -wave contributions in the first and third lines if the neutrino Yukawa couplings h_{ij}^N have the phases such as $\varphi_1 \neq \varphi_2 + n\pi$.¹⁴ This happens since the coannihilation cross section $\sigma_{\psi_{N_1}\psi_{N_2}}$ allows the s -wave contribution [7].

On the other hand, the s -wave contribution to the annihilation cross section of the bino-like χ to $\bar{f}f$ is expressed as [38]

$$\sigma_{\chi\chi} v \simeq \sum_f \frac{4c_f}{\pi} \frac{G_F^2 m_\chi^2 m_W^4 \beta'}{(m_0^2 + \beta'^2 m_\chi^2)^2} \tan^4 \theta_W \left((T_{3L} - Q_f)^4 + Q_f^4 + 2(T_{3L} - Q_f)^2 Q_f^2 \right) z^2, \quad (33)$$

where $z = m_f/m_\chi$, $\beta' = (1 - z^2)^{1/2}$ and Q_f is the electric charge of f . In the summation in eq. (33), f runs among quarks ($c_f = 3$) and leptons ($c_f = 1$) which satisfy $m_f < m_\chi$. Although $\chi\chi$ can annihilate to a pair of Higgs scalars through the Higgsino exchange, we neglect it as a subdominant process. If the bino mass $M_{\tilde{B}}$ is almost degenerate with Higgsino mass μ_H , the coannihilation between χ and a Higgsino-like neutralino χ' brings important effects on the relic abundance of χ . The relevant cross sections to

¹⁴The s -wave contributions are dominant in such cases. The expressions for $p_{F,S}$ for the corresponding p -wave contributions are shown in Appendix C for the completeness.

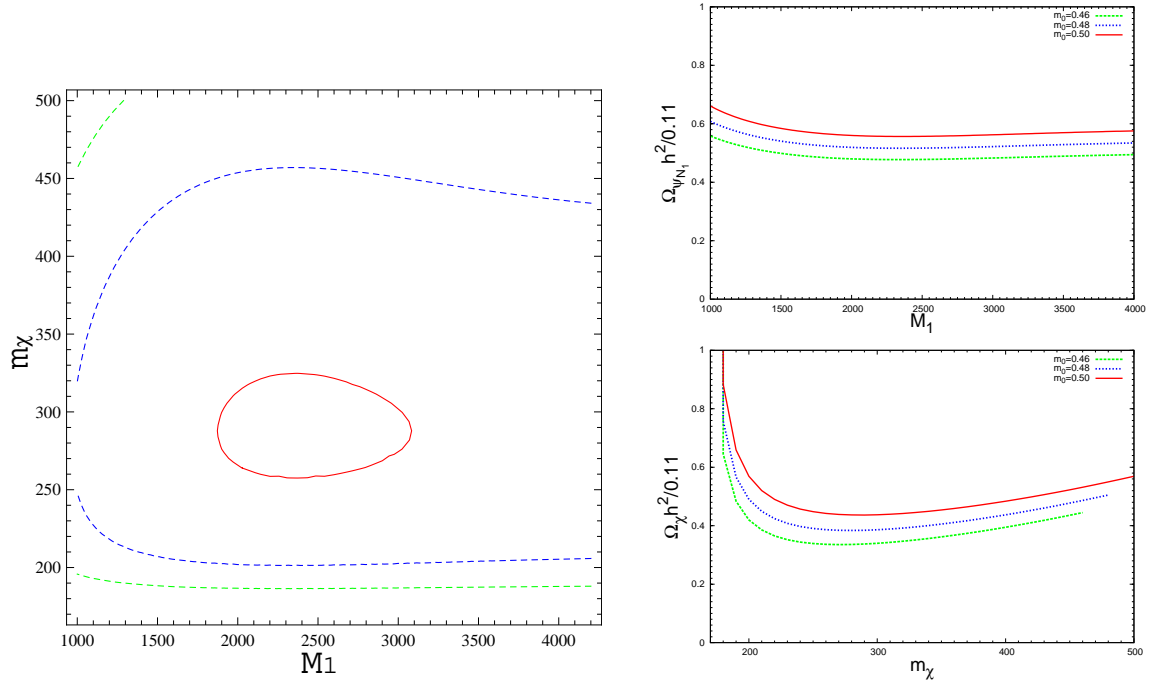


Fig. 5 The left frame shows the contours of $\Omega_{\psi_{N_1}} h^2 + \Omega_{\chi} h^2 = 0.11$ for $m_0 = 0.46$ TeV (a green dotted line), 0.48 TeV (a blue dotted line), and 0.50 TeV (a red solid line). Each figure in the right frames shows the ratio of each DM component to the total relic abundance $\Omega_{\psi_{N_1}} h^2 / 0.11$ and $\Omega_{\chi} h^2 / 0.11$, respectively. Each line corresponds to the same value of m_0 used in the left frame. In these figures $\varphi_1 - \varphi_2 = 0$ is assumed.

this coannihilation are given by [38]

$$\begin{aligned} \sigma_{\chi\chi'v} &\simeq \sum_f \frac{2c_f}{\pi} G_F^2 m_{\chi'}^2 \beta' \left[\frac{(g' \cos 2\beta / g \sin \beta) m_W m_f \xi_f}{[(m_{H_3}^2 - 4m_{\chi}^2)^2 + m_{H_3}^2 \Gamma_{H_3}^2]^{1/2}} - \frac{(T_{3L} \tan \theta_W) m_W m_f \zeta_f}{m_0^2 + \beta'^2 m_{\chi}^2} \right]^2, \\ \sigma_{\chi'\chi'v} &\simeq \sum_f \frac{4c_f}{\pi} G_F^2 m_{\chi'}^2 \beta' \left[\frac{(\xi_f \cot \beta / 2)(m_{\chi'} + \mu_H) m_f}{[(m_{H_3}^2 - 4m_{\chi}^2)^2 + m_{H_3}^2 \Gamma_{H_3}^2]^{1/2}} - \frac{m_f^2 \zeta_f^2 z}{4(m_0^2 + \beta'^2 m_{\chi}^2)} \right]^2, \end{aligned} \quad (34)$$

where $\xi_f = \cot \beta$ and $\zeta_f = \frac{1}{\sin \beta}$ for f in the up sector and $\xi_f = \tan \beta$ for and also $\zeta_f = \frac{1}{\cos \beta}$ for f in the down sector. The effective cross section including the coannihilation can be determined by the similar formulas to eq. (30). The dominant contributions for it are expected to come from a channel with the $t\bar{t}$ final state if $m_{\chi} > m_t$ is satisfied. If we focus our numerical study to the case with $m_{\chi} \simeq m'_{\chi}$ and fix both $\tan \beta$ and $m_{H_3}^2$ to suitable values¹⁵, these cross sections are determined by two free parameters m_0 and m_{χ} . They should satisfy some required conditions. Since χ should be lighter than the left-handed

¹⁵In this study we assume a sufficiently large value for $m_{H_3}^2$ such as $m_{H_3}^2 \gg 4m_{\chi}^2$, for simplicity.

sneutrinos, $m_\chi < m_0$ has to be satisfied. The new contribution to the $\mu \rightarrow e\gamma$ imposes the lower bound on m_0 as found from Fig. 3. In relation to this constraint it is useful to remind that a larger M_3 allows smaller values for m_0 .

Now we examine the possibility to realize the required relic abundance by these two DM. If we follow the ordinary method given in [37, 38], we can estimate the relic abundance by using the results for the effective annihilation cross section given above. Each relic abundance $\Omega_{\psi_{N_1}}$ and Ω_χ is given by the formulas

$$\Omega h^2 = \frac{1.07 \times 10^9 x_f}{g_*^{1/2} m_{\text{pl}} (\text{GeV}) (a_{\text{eff}} + 3b_{\text{eff}}/x_f)}, \quad x_f = \ln \frac{0.038 g_{\text{eff}} m_{\text{pl}} m_{DM} (a_{\text{eff}} + 6b_{\text{eff}}/x_f)}{g_*^{1/2} x_f^{1/2}}, \quad (35)$$

where $m_{\text{pl}} = 1.22 \times 10^{19}$ GeV and m_{DM} is the mass of DM. g_{eff} stands for the internal degrees of freedom of DM or the effective degrees of freedom in the coannihilation case. We can use $g_* \simeq 100$ as the relativistic degrees of freedom at the DM freeze-out temperature $T_f (\equiv m_{DM}/x_f)$ for both ψ_{N_1} and χ .

The results are shown in Fig. 5 for the parameter set used in the previous section. In the left frame, we plot the contours for the relic abundance (29) in the (M_1, m_χ) plane for $m_0 = 0.46$ TeV (a green dotted line), 0.48 TeV (a blue dotted line), and 0.50 TeV (a red solid line). Since we are considering that χ is the DM lighter than ψ_{N_1} , $M_1, m_0 > m_\chi$ should be satisfied. The allowed regions in the (M_1, m_χ) plane are represented by the points on each contour which satisfy this condition. They are found to have the almost fixed values of m_χ for the larger values of M_1 . The reason can be found in the right frame of Fig. 5, where the relic abundance of each DM component is plotted for the same parameters as the left frame. The same lines are used as the ones for the corresponding contours in the left frame. Since $\Omega_{\psi_{N_1}}$ is almost constant at large M_1 regions, the condition (29) can be satisfied only for restricted values of Ω_χ . Since both DM components have the same order abundance, we can expect rather different DM phenomenology from the one component DM models.

We need to note that the above solutions are obtained for rather large neutrino Yukawa couplings such as $\sqrt{|h_1|^2 + |h_2|^2} \simeq 3.56$ in case of $m_0 = 0.48$ TeV, for example. Although these results on the neutrino masses and the DM are interesting enough for the model, such large Yukawa couplings are dangerous for the perturbativity and the stability of the model [17]. However, this point can be improved by considering the case $\varphi_1 - \varphi_2 \neq n\pi$, which makes the s -wave contributions to the ψ_{N_1} annihilation cross section possible. Since

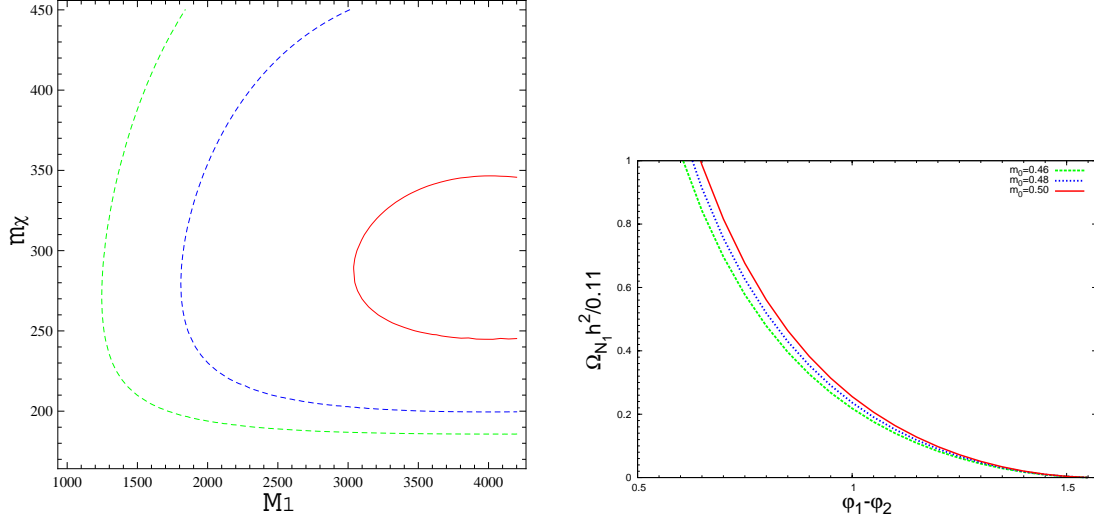


Fig. 6 The left frame is the same one as the left frame in Fig. 5. However, $\varphi_1 - \varphi_2 = \pi/4$ is assumed here. The right frame shows the phase dependence of $\Omega_{N_1} h^2$ for each value of m_0 in the left frame.

the s -wave contributions give much larger effects than the p -wave ones for the thermally averaged annihilation cross section $\langle\sigma v\rangle$, the neutrino Yukawa couplings required to reduce the relic abundance of ψ_{N_1} can be smaller. In the left frame of Fig. 6, the contours corresponding to the ones in the left frame in Fig. 5 are plotted by assuming $|h_1| = |h_2|$ and $\varphi_1 - \varphi_2 = \pi/4$. The parameters are fixed to the same values as in Fig. 5 except for $\bar{\lambda}$, which is taken here as $\bar{\lambda} = 8.4 \times 10^{-9}$. Since the neutrino Yukawa couplings have much smaller values like $\sqrt{|h_1|^2 + |h_2|^2} \simeq 1.45$ in case of $m_0 = 0.48$ TeV, the above mentioned tension is relaxed. In the right frame of Fig. 6, the phase dependence of $\Omega_{N_1} h^2$ is shown for the same parameter setting.

3 Probing two dark matter

The present model has two DM components, that is, the meta stable lightest singlet fermion ψ_{N_1} and the lightest neutralino χ . They are expected to be observed by several kinds of experiments. The former one may be studied indirectly through the decay products such as charged particles and gamma rays. The anomaly reported in the cosmic rays by PAMELA and Fermi-LAT may be relevant to this decay. The latter one may be observed directly through the elastic scattering with nuclei in the same way as the ordinary lightest superparticle. However, the situation can be rather different from the

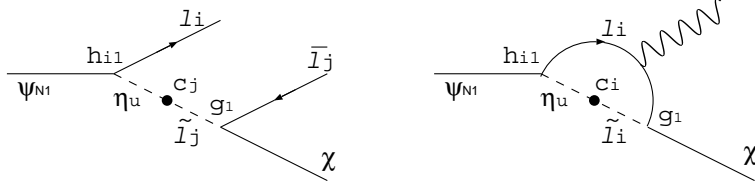


Fig. 7 Decay processes of ψ_{N_1} to χ . A bulb represents the anomaly induced interaction $c_j B M_{\text{pl}} e^{-b_j} \tilde{L}_j \tilde{\eta}_u$.

MSSM since the DM relic abundance is composed of two components. In this section these subjects are briefly discussed and the detailed study will be presented elsewhere.

3.1 Decay of the right-handed neutrino dark matter

The relic DM in this model can be composed of the two components ψ_{N_1} and χ as shown in the previous part. However, one of the DM candidates ψ_{N_1} is not stable since the Z_2 symmetry which guarantees its stability is not exact. It should be reminded that this symmetry is the remnant symmetry left after the spontaneous breaking of the anomalous $U(1)_X$. Green-Schwarz anomaly cancellation mechanism induces the Z_2 violating interaction and also the corresponding soft supersymmetry breaking term, which are shown in eqs. (9) and (10). If ψ_{N_1} is heavier than χ , this interaction brings the decay of ψ_{N_1} to χ through the diagrams shown in Fig. 7.

We can estimate the lifetime of ψ_{N_1} due to the decay derived by this interaction. It can be expressed as

$$\tau_{\psi_{N_1}} \sim \left(\frac{1 \text{ TeV}}{M_1} \right)^5 \left(\frac{\mu_\eta}{1.3 \text{ TeV}} \right)^4 \left(\frac{m_0}{1 \text{ TeV}} \right)^4 \left(\frac{1 \text{ TeV}}{B} \right)^2 \left(\frac{e^{2b_j}}{10^{80}} \right) \times 10^{26} \text{ sec}, \quad (36)$$

where we use $h_{i1}^N = O(1)$ and $c_j = O(1)$. If $M_1 \gg m_0$ is satisfied, m_0 should be replaced by M_1 in eq. (36). From this formula, we find that ψ_{N_1} can have a sufficiently long lifetime compared with the age of universe as long as $b_j > 82$ is satisfied. Thus, although the true stable DM is the lightest neutralino χ , we need to take account of the contribution of ψ_{N_1} to the relic DM abundance in the universe as discussed in the previous part.

On the other hand, depending on the scale of the Z_2 breaking $c_j M_{\text{pl}} B e^{-b_j}$ in eq. (10), particles yielded in the decay of ψ_{N_1} could bring additional contributions to the cosmic rays in the present universe. They could be detected as the anomaly in the expected flux of the cosmic rays through the various observation. In fact, if $b_j \sim 92$ is satisfied, this anomaly induced superweak interaction causes a large enhancement factor of $O(10^{80})$

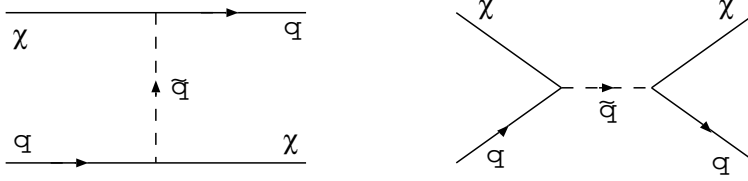


Fig. 8 Elastic scattering diagrams of the bino-like neutralino χ with the quark which are relevant to the direct search of χ .

in eq. (36) to realize a long lifetime of $O(10^{26})$ sec for ψ_{N_1} .¹⁶ This lifetime is known to be suitable to explain the charged cosmic ray anomaly reported by PAMELA [12] and Fermi-LAT [13]. Since ψ_{N_1} couples only with leptons and sleptons because of the $U(1)_X$ symmetry, this decay can yield a pair of a lepton and an antilepton, or a photon in addition to the lightest neutralino χ . This feature is favored by the lepto-philic nature of the PAMELA observations. Moreover, the flavor structure of the neutrino Yukawa couplings (17) can restrict the flavor of the final charged leptons to μ and τ .¹⁷ Since this makes positrons and electrons produced by their decay soft, the model becomes favorable for the explanation of a plateau at high energy regions of $e^+ + e^-$ spectrum found in the Fermi-LAT observations [39].

The heavier DM component ψ_{N_1} has also a radiative decay mode to the lightest neutralino χ . Its one-loop diagram is shown in Fig. 7. This decay associates a characteristic gamma which can be detected through the observation of the cosmic gamma rays. It is expected to appear as a line shape spectrum at the energy $E_\gamma = (M_1^2 - m_\chi^2)/2M_1$, which corresponds to the endpoint of the gamma-ray spectrum caused by the bremsstrahlung and the inverse Compton scatterings associated to the charged decay products of ψ_{N_1} . This could be a clear evidence of the model.

3.2 Direct detection of the neutralino dark matter

Direct detection of the DM is expected to clarify the nature of DM [3]. Several experiments such as CDMSII, XENON100 and XMASS to observe its elastic scattering with nuclei are

¹⁶It is interesting to note that this value of b_j can be consistent with the value of $\text{Tr}X$ for $g_X = O(1)$, which is required to realize the vacuum with the desired values of ε_\pm as discussed below eq. (5) and in Appendix A.

¹⁷In that case we need to impose $c_e = 0$ additionally in the anomaly induced interaction $c_j M_{\text{Pl}} e^{-b_j} L_j \eta_u$.

now under going or will start in the near future. In the study of DM models, it is crucial to address the discriminative features of the model, which are expected to be shown in these experiments.

Since one DM component ψ_{N_1} interacts with the leptons only and can not have interactions with nuclei at tree level, the scattering cross section with nuclei is heavily suppressed by the loop factor. Thus, it is difficult to detect it in these experiments. On the other hand, the neutralino DM χ can be scattered with nuclei at tree level since it has the same nature as an ordinary neutralino in the MSSM. One may consider that there is no distinction with the MSSM case. However, it should be noted that the detection rate R in this model can be different from the direct detection rate R_{MSSM} in the MSSM even if the DM χ has the same mass and the same scattering cross section with a nucleon in two models. The detection rate is approximately defined by

$$R \simeq \sum_i n_i \frac{\rho_\chi}{m_\chi} \langle \sigma_{i\chi} \rangle, \quad (37)$$

where n_i is the number of i nuclei species in the detector and $\langle \sigma_{i\chi} \rangle$ is the scattering cross section of the χ and i nuclei species averaged over the relative velocity between the χ and the detector. Since the present model has two DM components, the detection rate in this model is related to the MSSM one as

$$R = \frac{\Omega_\chi}{\Omega_{\psi_{N_1}} + \Omega_\chi} R_{\text{MSSM}}, \quad (38)$$

for the fixed m_χ and m_0 . Therefore, in the present model the parameter regions to realize the same DM detection rate can be changed from the ones in the MSSM, although the interactions of the χ with quarks are same as the MSSM neutralino. This might open a new possibility for the supersymmetry breaking parameters, which is considered not to be allowed in the MSSM case.

In order to see this aspect we consider the bino-like χ , as an example. The dominant contributions to the elastic scattering of the bino-like χ with nuclei come from the squark exchange shown in Fig. 8. In the MSSM case, the allowed parameter regions relevant to the detection rate have been extensively studied. We are interested here in possible changes of the detection rate from the MSSM and also the change of the allowed region of the supersymmetry breaking parameters.

We suppose the values used in Fig. 5 for the parameters relevant to the neutrino mass generation. Eq. (38) shows that the figure of R/R_{MSSM} as a function of m_χ is the same

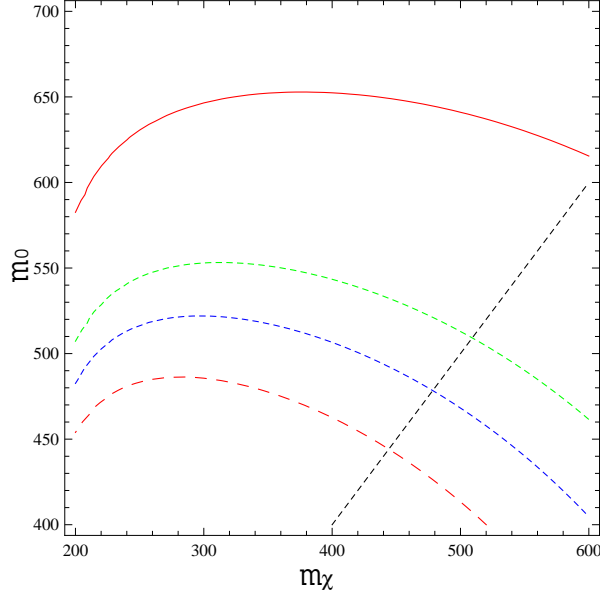


Fig. 9 The Contours of R/R_{MSSM} in the (m_χ, m_0) plane. Each line corresponds to $R/R_{\text{MSSM}} = 1$ (a red solid line), 0.6 (a green dotted line), 0.5 (a blue dotted line) and 0.4 (a red dashed line). A thin black dashed line represents a line for $m_0 = m_\chi$.

as the one in the right frames of Fig. 5 for the case $\varphi_1 - \varphi_2 = 0$. If we considered the case with $m_0 = 0.48$ TeV and $M_1 = 2.5$ TeV as an example, Fig. 5 shows that m_χ should be 0.2 TeV or 0.45 TeV to satisfy the WMAP constraint and then $R/R_{\text{MSSM}} \simeq 0.5$ or 0.47 for each m_χ value, respectively. These suppressed detection rates compared with the MSSM are realized for much smaller m_0 values than the ones in the MSSM. The values of (m_χ, m_0) found through the direct search of the lightest neutralino are placed on the contours of $\Omega_\chi h^2$, which exist in the region where $\Omega_\chi h^2 < 0.11$, $m_0 > m_\chi$ and $m_0 > 0.45$ TeV. The last condition comes from the $\mu \rightarrow e\gamma$ constraint in Fig. 3. We note that the model could bring new possibilities for the detection rate of the neutralino DM and values of (m_χ, m_0) relevant to the supersymmetry breaking even if we impose the DM relic abundance constraint (see Fig. 9).

4 Summary

We have studied a supersymmetric model with an anomalous U(1) gauge symmetry from a view point of the neutrino masses and the DM. The model considered in this paper may be recognized as a supersymmetric extension of the radiative seesaw model for the

neutrino masses proposed by Ma. The Froggatt-Nielsen mechanism based on the spontaneously broken anomalous $U(1)$ symmetry generates the hierarchical structure of Yukawa couplings of the quarks and the charged leptons. Moreover, it can also explain the hierarchical couplings and mass scales required for the radiative generation of the neutrino masses. If we assume a flavor structure for the neutrino Yukawa couplings, the tri-bimaximal mixing is automatically induced.

The model has two DM components. One is stable and the other is the decaying DM. Their stability is guaranteed by two Z_2 symmetries, one of which is the ordinary R parity. Since one of these discrete symmetries is assumed to be anomalous and then the Z_2 violating interaction is generated nonperturbatively, the instability of one DM component is caused. This phenomenon happens since this Z_2 symmetry is embedded into the anomalous $U(1)$ gauge symmetry. Since the Z_2 violating nonperturbative interaction is extremely weak, the huge suppression factor for the decay width of this DM can be derived. As a result, its lifetime becomes longer than the age of the universe. These DM particles can be detected through both direct and indirect searches. Thus, the model may be checked through these experiments. In particular, the recent and future data coming from the cosmic ray observations might clarify the relation between this model and the physics beyond the SM.

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Appendix A

In this appendix we give two examples of the $U(1)_X$ charge assignment which induce the favorable effective parameters for the generation of the mass eigenvalues and mixings of the quarks and the leptons including the neutrinos. If we define ε_{\pm} as $\varepsilon_{\pm} = \langle \Sigma_{\pm} \rangle / M_{\text{pl}}$, eq. (7) gives the expression for each parameter in the superpotential W such as

$$\begin{aligned} h_{ij}^U &= y_{ij}^U \varepsilon_-^{n_{Q_i} + n_{U_j} + n_{H_u}}, & h_{ij}^D &= y_{ij}^D \varepsilon_-^{n_{Q_i} + n_{D_j} + n_{H_d}}, & h_{ij}^E &= y_{ij}^E \varepsilon_-^{n_{L_i} + n_{E_j} + n_{H_d}}, \\ \mu_H &= \lambda_H M_{\text{pl}} \varepsilon_+^{-\frac{n_{H_u} + n_{H_d}}{n_+}}, \end{aligned} \quad (39)$$

for the ones belonging to the MSSM. Other ones are also expressed as

$$\begin{aligned} h_{ij}^N &= y_{ij}^N \varepsilon_-^{n_{L_i} + n_{N_j} + n_{\eta_u} + 1}, & \lambda_u &= y_{\eta_u} \varepsilon_+^{-\frac{n_{\eta_u} + n_{\phi} + n_{H_d} + 1}{n_+}}, & \lambda_d &= y_{\eta_d} \varepsilon_+^{-\frac{n_{\eta_d} + n_{\phi} + n_{H_u} + 1}{n_+}}, \\ \mu_{\eta} &= y_{\eta} M_{\text{pl}} \varepsilon_+^{-\frac{n_{\eta_u} + n_{\eta_d} + 1}{n_+}}, & M_{ij} &= y_{N_i N_j} M_{\text{pl}} \varepsilon_+^{-\frac{n_{N_i} + n_{N_j} + 1}{n_+}}, & \mu_{\phi} &= y_{\phi} M_{\text{pl}} \varepsilon_-^{2n_{\phi} + 1}. \end{aligned} \quad (40)$$

In these formulas, it may be natural to suppose that the original coupling constants in the nonrenormalizable interactions, that is, y_{ij}^U , λ_H and so on, have values of $O(1)$.

We assume that Σ_- and Σ_+ obtain the VEVs defined by $\varepsilon_- \simeq 10^{-1}$ and $\varepsilon_+ \simeq 10^{-4}$, respectively. The $U(1)_X$ charge is assigned to each field as follows,

$$\begin{aligned} n_Q &= (6, 5, 3), & n_U &= (6, 5, 3), & n_D &= (4, 3, 3), \\ \text{example (i)} \quad n_L &= (9, 9, 9), & n_E &= (1, -1, -3), & n_{H_u} &= n_{H_d} = -6, \\ n_N &= (-5, -5, -5), & n_{\eta_u} &= n_{\eta_d} = -5, & n_{\phi} &= 7, & n_+ &= 3. \end{aligned}$$

In this case, it is easily found that the mass matrices for the up- and down-type quarks and the charged leptons take the following form:

$$\begin{aligned} M_U &= \begin{pmatrix} \varepsilon_-^6 & \varepsilon_-^5 & \varepsilon_-^3 \\ \varepsilon_-^5 & \varepsilon_-^4 & \varepsilon_-^2 \\ \varepsilon_-^3 & \varepsilon_-^2 & 1 \end{pmatrix} \langle H_u^0 \rangle, & M_D &= \begin{pmatrix} \varepsilon_-^4 & \varepsilon_-^3 & \varepsilon_- \\ \varepsilon_-^3 & \varepsilon_-^2 & 1 \\ \varepsilon_-^3 & \varepsilon_-^2 & 1 \end{pmatrix} \langle H_d^0 \rangle, \\ M_E &= \begin{pmatrix} \varepsilon_-^4 & 0 & 0 \\ 0 & \varepsilon_-^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \langle H_d^0 \rangle, \end{aligned} \quad (41)$$

where these M_f are defined as $\bar{\psi}_R M_f \psi_L$. In the charged lepton mass matrix, the off-diagonal couplings are supposed to satisfy $y_{ij}^E = 0$ (for $i \neq j$). From these mass matrices

we obtain both the mass eigenvalues and the CKM matrix in the quark sector as

$$\begin{aligned} m_u : m_c : m_t &= \varepsilon_-^6 : \varepsilon_-^4 : 1, & m_d : m_s : m_b &= \varepsilon_-^4 : \varepsilon_-^2 : 1, \\ V_{us} &\sim \varepsilon_-, & V_{ub} &\sim \varepsilon_-^3, & V_{cb} &\sim \varepsilon_-^2. \end{aligned} \quad (42)$$

The charged lepton mass eigenvalues satisfy

$$m_e : m_\mu : m_\tau = \varepsilon_-^4 : \varepsilon_-^2 : 1. \quad (43)$$

These can give qualitatively good results as long as ε_- takes a value of Cabibbo mixing angle 0.22. The effective neutrino Yukawa couplings have no hierarchical structure $h_{ij}^N = y_{ij}^N = O(1)$ as supposed in the text. Since the effective μ_H term is generated as $M_{\text{pl}}\varepsilon_+^4$, it can take an appropriate value for the electroweak symmetry breaking.

Other effective parameters are estimated as

$$\lambda_u \sim \lambda_d = O(\varepsilon_+), \quad M_i = O(y_{N_i N_i} M_{\text{pl}} \varepsilon_+^3), \quad \mu_\eta = O(y_\eta M_{\text{pl}} \varepsilon_+^3), \quad \mu_\phi = O(M_{\text{pl}} \varepsilon_-^{15}), \quad (44)$$

where we suppose that the off-diagonal couplings $y_{N_i N_j}$ are zero. These parameters are intimately related to the neutrino mass generation in the present model. The values of these parameters used in the text can be realized if $y_{N_i N_i}$ and y_η have suppressed values of $O(10^{-2})$.

In the above example, we implicitly assume that each parameter is determined by either ε_- or ε_+ only but is not determined by both of them. In the next example, we consider that some of them are determined by both ε_- and ε_+ . We assume that Σ_- and Σ_+ obtain the VEVs defined by $\varepsilon_- \simeq 10^{-1}$ and $\varepsilon_+ \simeq 4 \times 10^{-4}$ again. The $U(1)_X$ charge is assigned to each field as follows,

$$\begin{aligned} n_Q &= (6, 5, 3), & n_U &= (3, 2, 0), & n_D &= (5, 4, 4), \\ \text{example (ii)} \quad n_L &= (7, 7, 7), & n_E &= (4, 2, 0), & n_{H_u} &= -3, & n_{H_d} &= -7, \\ n_N &= (-6, -6, -6), & n_{\eta_u} &= -2, & n_{\eta_d} &= -9, & n_\phi &= 6, & n_+ &= 3. \end{aligned} \quad (45)$$

Using these charges, we can easily find what kind of factor determined by ε_- and ε_+ should appear as the lowest order one for each term in W . The mass matrices of quarks and charged leptons and also the neutrino Yukawa couplings h_{ij}^N show the same features as the ones in the previous example.

The remaining effective parameters are estimated as

$$\begin{aligned} \mu_H &= O(M_{\text{pl}} \varepsilon_+^4 \varepsilon_-^2), & \lambda_u &= O(\varepsilon_+ \varepsilon_-), & \lambda_d &= O(\varepsilon_+^2 \varepsilon_-), \\ M_i &= O(M_{\text{pl}} \varepsilon_+^3 \varepsilon_-), & \mu_\eta &= O(M_{\text{pl}} \varepsilon_+^4 \varepsilon_-^2), & \mu_\phi &= O(M_{\text{pl}} \varepsilon_-^{13}). \end{aligned} \quad (46)$$

These also result in the favorable values for the parameters relevant to neutrino mass generation and also the electroweak symmetry breaking. However, it is useful to note a following point. Since $\lambda_u \gg \lambda_d$ is satisfied in this case, the formula for the dominant contribution to the neutrino masses can be changed. In fact, the term with λ_u^2 in eq. (47) could cause the similar order contribution as eq. (15) or larger contribution even if there is a suppression due to the large μ_ϕ .

Appendix B

In this Appendix we give the formulas for the contributions to the neutrino masses due to the diagrams (b) and (c) in Fig. 1. These diagrams have the scalar component and the fermion component of the chiral superfield ϕ as an internal line, respectively. The expressions for their contributions can be derived by taking account of eq. (14).

The neutrino mass matrix induced by the diagram (b) can be expressed as

$$\begin{aligned}
(\mathcal{M}_\nu)_{\alpha\beta} = & \sum_{a=\pm} \sum_{i=1}^3 \frac{h_{\alpha i} h_{\beta i} M_i (A + a\mu_\phi)^2 a}{(4\pi)^2} \\
& \times \left[\frac{\lambda_d^2 v_u^2}{8} \left(I(m_{\eta+}, m_{\eta+}, M_i, m_{\phi a}) + I(m_{\eta-}, m_{\eta-}, M_i, m_{\phi a}) - 2I(m_{\eta+}, m_{\eta-}, M_i, m_{\phi a}) \right) \right. \\
& + \frac{\lambda_u^2 v_d^2}{8} \left(I(m_{\eta+}, m_{\eta+}, M_i, m_{\phi a}) + I(m_{\eta-}, m_{\eta-}, M_i, m_{\phi a}) + 2I(m_{\eta+}, m_{\eta-}, M_i, m_{\phi a}) \right) \\
& \left. + \frac{\lambda_u \lambda_d v_u v_d a}{4} \left(I(m_{\eta+}, m_{\eta+}, M_i, m_{\phi a}) - I(m_{\eta-}, m_{\eta-}, M_i, m_{\phi a}) \right) \right], \tag{47}
\end{aligned}$$

where the function I is defined as

$$\begin{aligned}
I(m_a, m_b, m_c, m_d) &= \frac{m_a^2 \ln m_a^2}{(m_b^2 - m_a^2)(m_c^2 - m_a^2)(m_d^2 - m_a^2)} + \frac{m_b^2 \ln m_b^2}{(m_c^2 - m_b^2)(m_d^2 - m_b^2)(m_a^2 - m_b^2)} \\
&+ \frac{m_c^2 \ln m_c^2}{(m_d^2 - m_c^2)(m_a^2 - m_c^2)(m_b^2 - m_c^2)} + \frac{m_d^2 \ln m_d^2}{(m_a^2 - m_d^2)(m_b^2 - m_d^2)(m_c^2 - m_d^2)}, \\
I(m_a, m_a, m_c, m_d) &= \frac{(m_a^4 - m_c^2 m_d^2) \ln m_a^2}{(m_c^2 - m_a^2)^2 (m_d^2 - m_a^2)^2} + \frac{m_c^2 \ln m_c^2}{(m_d^2 - m_c^2)(m_a^2 - m_c^2)^2} \\
&+ \frac{m_d^2 \ln m_d^2}{(m_c^2 - m_d^2)(m_a^2 - m_d^2)^2} - \frac{1}{(m_c^2 - m_a^2)(m_d^2 - m_a^2)}. \tag{48}
\end{aligned}$$

On the other hand, the neutrino mass matrix induced by the diagram (c) is estimated

as

$$\begin{aligned}
(\mathcal{M}_\nu)_{\alpha\beta} &= \frac{\lambda_d^2 v_u^2 \mu_\eta^2 \mu_\phi}{2(4\pi)^2} \sum_{a=\pm} \sum_{i=1}^3 h_{\alpha i} h_{\beta i} I(\mu_\eta, \mu_\eta, M_{ia}, \mu_\phi) \\
&- \frac{\lambda_u^2 v_d^2 \mu_\phi}{(4\pi)^2} \sum_{a=\pm} \sum_{i=1}^3 h_{\alpha i} h_{\beta i} J(\mu_\eta, M_{ia}, \mu_\phi),
\end{aligned} \tag{49}$$

where J is defined as

$$\begin{aligned}
J(m_a, m_b, m_c) &= \left(-\frac{m_a^2}{(m_b^2 - m_a^2)(m_c^2 - m_a^2)} + \frac{m_a^4(2m_a^2 - m_b^2 - m_c^2)}{2(m_b^2 - m_a^2)^2(m_c^2 - m_a^2)^2} \right) \ln m_a^2 \\
&+ \frac{m_b^4 \ln m_b^2}{2(m_c^2 - m_b^2)(m_a^2 - m_b^2)^2} + \frac{m_c^4 \ln m_c^2}{2(m_b^2 - m_c^2)(m_a^2 - m_c^2)^2} \\
&- \frac{m_a^2}{2(m_b^2 - m_a^2)(m_c^2 - m_a^2)}.
\end{aligned} \tag{50}$$

As long as $\mu_\phi \gg \mu_\eta, M_i, m_0, A$ and $\lambda_u \simeq \lambda_d$ are satisfied, these contributions are found to be subdominant in comparison with the one induced by the diagram (a) except for the ones proportional to μ_ϕ^2 in eq. (47). These cause non-negligible contributions to eq. (15), which are estimated as

$$\begin{aligned}
&\sum_{i=1}^3 \frac{h_{\alpha i} h_{\beta i} M_i}{(4\pi)^2} \left[\frac{\lambda_d^2 v_u^2}{4} \left(\tilde{I}(m_{\eta+}, m_{\eta+}, M_i) + \tilde{I}(m_{\eta-}, m_{\eta-}, M_i) - 2\tilde{I}(m_{\eta+}, m_{\eta-}, M_i) \right) \right. \\
&\quad \left. + \frac{\lambda_u^2 v_d^2}{4} \left(\tilde{I}(m_{\eta+}, m_{\eta+}, M_i) + \tilde{I}(m_{\eta-}, m_{\eta-}, M_i) + 2\tilde{I}(m_{\eta+}, m_{\eta-}, M_i) \right) \right],
\end{aligned} \tag{51}$$

where the function \tilde{I} is defined as

$$\begin{aligned}
\tilde{I}(m_a, m_b, m_c) &= \frac{m_a^2 \ln m_a^2}{(m_b^2 - m_a^2)(m_c^2 - m_a^2)} + \frac{m_b^2 \ln m_b^2}{(m_c^2 - m_b^2)(m_a^2 - m_b^2)} + \frac{m_c^2 \ln m_c^2}{(m_b^2 - m_c^2)(m_a^2 - m_c^2)}, \\
\tilde{I}(m_a, m_a, m_c) &= \frac{m_a^2 - m_c^2 + m_c^2 \ln(m_c^2/m_a^2)}{(m_a^2 - m_c^2)^2}.
\end{aligned} \tag{52}$$

This could be the same order contribution to the neutrino mass matrix as the one given in eq.(15). However, it is useful to note that these contributions do not change the structure of the MNS matrix but they only change the values of Λ_i somewhat.

Appendix C

The p -wave contributions in case of $\varphi_1 \neq \varphi_2 + n\pi$ are given by

$$\begin{aligned}
p_F &= \frac{1}{3} - \frac{M_1^2(3M_1^2 + 5m_{\eta a}^2)}{6(M_1^2 + m_{\eta a}^2)^2} - \frac{M_1^2(3M_1^2 + 5m_{\eta b}^2)}{6(M_1^2 + m_{\eta b}^2)^2} + \frac{M_1^4}{3(M_1^2 + m_{\eta a}^2)(M_1^2 + m_{\eta b}^2)}, \\
p_S &= -\frac{1}{2} + \frac{5}{12}\beta^2 + \frac{M_1^2\beta^2(\mu_\eta^2 + 4M_1^2\beta^2)}{3(\mu_\eta^2 + M_1^2\beta^2)^2}.
\end{aligned} \tag{53}$$

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